Opportunistic Network Decoupling With Virtual Full-Duplex Operation in Multi-Source Interfering Relay Networks

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Abstract—We introduce a new achievability scheme, termed *opportunistic network decoupling (OND)*, operating in virtual full-duplex mode. In the scheme, a novel relay scheduling strategy is utilized in the $K \times N \times K$ *channel with interfering relays*, consisting of K source–destination pairs and N half-duplex relays in-between them. A subset of relays using alternate relaying is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. As our main result, it is shown that under a certain relay scaling condition, the OND protocol achieves K degrees of freedom even in the presence of interfering links among relays. Numerical evaluation is also shown to validate the performance of the proposed OND. Our protocol basically operates in a fully distributed fashion along with local channel state information, thereby resulting in a relatively easy implementation.

Index Terms—Degrees of freedom (DoF), half-duplex, interference, $K \times N \times K$ channel, opportunistic network decoupling (OND), relay, virtual full-duplex (FD).

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1 INTRODUCTION

1.1 Previous Work

INTERFERENCE between wireless links has been taken into account as a critical problem in wireless communication systems. Recently, interference alignment (IA) was proposed for fundamentally solving the interference problem when there are two communication pairs [1]. It was shown in [2] that the IA scheme can achieve the optimal degrees of freedom (DoF), which is equal to $K/2$, in the K-user interference channel with time-varying channel coefficients. Since then, interference management schemes based on IA have been further developed and analyzed in various wireless network environments: multiple-input multiple-output (MIMO) interference networks [3], [4], X networks [5], and cellular networks [6], [7], [8], [9].

On the one hand, following up on these successes for single-hop networks, more recent and emerging work

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has studied multihop networks with multiple source-destination (S–D) pairs. For the 2-user 2-hop network with 2 relays (referred to as the $2 \times 2 \times 2$ interference channel), it was shown in [10] that interference neutralization combining with symbol extension achieves the optimal DoF. A more challenging network model is to consider K-user two-hop relay-aided interference channels, consisting of K source-destination (S–D) pairs and N helping relay nodes located in the path between S–D pairs, so-called the $K \times N \times K$ channel. Several achievability schemes have been known for the network, but more detailed understanding is still in progress. By applying the result from [11] to the $K \times N \times K$ channel, one can show that $K/2$ DoF is achieved by using orthogonalize-and-forward relaying, which completely neutralizes interference at all destinations if N is greater than or equal to $K(K-1) + 1$. Another achievable scheme, called aligned network diagonalization, was introduced in [12] and was shown to achieve the optimal DoF in the $K \times N \times K$ channel while tightening the required number of relays. The scheme in [12] is based on the real interference alignment framework [7]. In [10], [12], however, the system model under consideration assumes that there is no interfering signal between relays and the relays are full-duplex. Moreover, in [13], the $2 \times 2 \times 2$ interference channel with full-duplex relays interfering with each other was characterized and its DoF achievability was shown using aligned interference

neutralization.[1]

On the other hand, there are lots of results on the usefulness of fading in the literature, where one can obtain the multiuser diversity gain in broadcast channels: opportunistic scheduling [15], opportunistic beamforming [16], and random beamforming [17]. Such opportunism can also be fully utilized in multi-cell uplink or downlink networks by using an opportunistic interference alignment strategy [9], [18], [19], [20]. Various scenarios exploiting the multiuser diversity gain have been studied in cooperative networks by applying an opportunistic two-hop relaying protocol [21] and an opportunistic routing [22], and in cognitive radio networks with opportunistic scheduling [23], [24]. In addition, recent results [25], [26] have shown how to utilize the opportunistic gain when there are a large number of channel realizations. More specifically, to amplify signals and cancel interference, the idea of opportunistically pairing complementary channel instances has been studied in interference networks [25] and multi-hop relay networks [26]. In cognitive radio environments [27], opportunistic spectrum sharing was introduced by allowing secondary users to share the radio spectrum originally allocated to primary users via transmit adaptation in space, time, or frequency.

1.2 Main Contributions

In this paper, we study the $K \times N \times K$ *channel with interfering relays*, which can be taken into account as one of practical multi-source interfering relay networks and be regarded as a fundamentally different channel model from the conventional $K \times N \times K$ channel in [12]. Then, we introduce an *opportunistic network decoupling (OND)* protocol that achieves full DoF with comparatively easy implementation under the channel model. This work focuses on the $K \times N \times K$ channel with one additional assumption that N *half-duplex* (HD) relays interfere with each other, which is a more feasible scenario. The scheme adopts the notion of the multiuser diversity gain for performing interference management over two hops. More precisely, in our scheme, a scheduling strategy is presented in time-division duplexing (TDD) two-hop environments with time-invariant channel coefficients, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level. To improve the spectral efficiency, the *alternate relaying* protocol in [28], [29], [30] is employed with a modification, which eventually enables our system to operate in *virtual full-duplex* mode. As our main result, it turns out that in a high signal-to-noise ratio (SNR) regime, the OND protocol asymptotically achieves the min-cut upper bound

1. The idea in [13] was later extended to the 2-user 3-hop network with 4 relays, i.e., the $2 \times 2 \times 2 \times 2$ interference channel [14].

of K DoF even in the presence of inter-relay interference and half-duplex assumption, provided the number of relays, N, scales faster than snr^{3K-2}, which is the minimum number of relays required to guarantee our achievability result. Numerical evaluation also indicates that the OND protocol has higher sum-rates than those of other relaying methods under realistic network conditions (e.g., finite N and SNR) since the inter-relay interference is significantly reduced owing to the opportunistic gain. For comparison, the OND scheme without alternate relaying and the max-min SNR scheme are also shown as baseline schemes. Note that our protocol basically operates with local channel state information (CSI) at the transmitter and thus is suitable for distributed/decentralized networks.

Our main contributions are fourfold as follows:

- In the $K \times N \times K$ channel with interfering relays, we introduce a new achievability scheme, termed OND with virtual full-duplex operation.
- Under the channel model, we completely analyze the optimal DoF, the required relay scaling condition, and the decaying rate of the interference level, where the OND scheme is shown to approach the min-cut upper bound on the DoF.
- Our achievability result (i.e., the derived DoF and relay scaling law) is validated via numerical evaluation.
- We perform extensive computer simulations with other baseline schemes.

1.3 Organization

The rest of this paper is organized as follows. In Section 2, we describe the system and channel models. In Section 3, the proposed OND scheme is specified and its lower bound on the DoF is analyzed. Section 4 shows an upper bound on the DoF. Numerical results of the proposed OND scheme are provided in Section 5. Finally, we summarize the paper with some concluding remarks in Section 6.

1.4 Notations

Throughout this paper, $\mathbb{C}$, $\mathbb{E}[\cdot]$, and $\lceil \cdot \rceil$ indicate the field of complex numbers, the statistical expectation, and the ceiling operation, respectively. Unless otherwise stated, all logarithms are assumed to be to the base 2.

Moreover, TABLE 1 summarizes the notations used throughout this paper. Some notations will be more precisely defined in the following sections, where we introduce our channel model and achievability results.

TABLE 1
Summary of notations

Notation	Description
$\mathcal{S}_k$	kth source
$\mathcal{D}_k$	kth destination
$\mathcal{R}_k$	kth relay
$h_{ik}^{(1)}$	channel coefficient from $\mathcal{S}_k$ to $\mathcal{R}_i$
$h_{ki}^{(2)}$	channel coefficient from $\mathcal{R}_i$ to $\mathcal{D}_k$
$h_{ik}^{(r)}$	channel coefficient between $\mathcal{R}_i$ and $\mathcal{R}_k$
$x_k^{(1)}(l)$	lth transmitted symbol of kth source
$\pi_s(k)$ $(s = 1, 2)$	indices of two relays helping kth S–D pair
$x_k^{(1)}(l)$	lth transmit symbol of $\mathcal{S}_k$
$x_{\pi_s(k)}^{(2)}(l)$	lth transmit symbol of $\mathcal{R}_{\pi_s(k)}$
$\mathbf{\Pi}_s$ $(s = 1, 2)$	two selected relay sets
$\tilde{L}_{i,k}$	scheduling metric in Step 1
$L_{i,k}^{\mathbf{\Pi}_2}$	scheduling metric in Step 2
$\mathsf{DoF}_{\text{total}}$	total number of DoF
$\mathsf{sinr}_{\pi_s(k)}^{(1)}$	SINR at $\mathcal{R}_{\pi_s(k)}$
$\mathsf{sinr}_{k,\pi_s(k)}^{(2)}$	SINR at $\mathcal{D}_k$ (from $\mathcal{R}_{\pi_s(k)}$)

2 SYSTEM AND CHANNEL MODELS

As one of two-hop cooperative scenarios, we consider the $K \times N \times K$ channel model with interfering relays, which fits into the case where each S–D pair is geographically far apart and/or experiences strong shadowing (thus requiring the response to a huge challenge for achieving the target spectral efficiency). In the channel model, it is thus assumed that each source transmits its own message to the corresponding destination only through one of N relays, and thus there is no direct path between an S–D pair. Note that unlike the conventional $K \times N \times K$ channel, relay nodes are assumed to interfere with each other in our model. There are K S–D pairs, where each receiver is the destination of exactly one source node and is interested only in traffic demands of the source. As in the typical cooperative relaying setup, N relay nodes are located in the path between S–D pairs so as to help to reduce path-loss attenuations.

Suppose that each node is equipped with a single transmit antenna. Each relay node is assumed to operate in half-duplex mode and to fully decode, re-encode, and retransmit the source message i.e., decode-and-forward protocol is taken into account. We assume that each node (either a source or a relay) has an average transmit power constraint P. Unlike the work in [10], [12], N relays are assumed to interfere with each other.[2] To improve the spectral efficiency, the alternate relaying protocol in [28], [29], [30] is employed

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2. If we can cancel the interfering signals among multiple relays, then the existing achievable scheme of the $K \times N \times K$ channel can also be applied here.
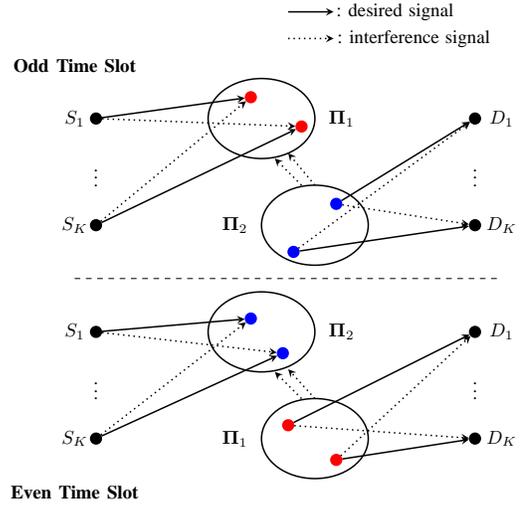

Fig. 1. The overall procedure of our OND scheme in the $K \times N \times K$ channel with interfering relays.

with a modification. With alternate relaying, each selected relay node toggles between the transmit and listen modes for alternate time slots of message transmission of the sources. If N is sufficiently large, then it is possible to exploit the channel randomness for each hop and thus to obtain the opportunistic gain in multiuser environments. In this work, we do not assume the use of any sophisticated multiuser detection schemes at each receiver (either a relay or a destination node), thereby resulting in an easier implementation.

Now, let us turn to channel modeling. Let $\mathcal{S}_k$, $\mathcal{D}_k$, and $\mathcal{R}_i$ denote the kth source, the corresponding kth destination, and the ith relay node, respectively, where $k \in \{1, \cdots, K\}$ and $i \in \{1, \cdots, N\}$. The terms $h_{ik}^{(1)}, h_{ki}^{(2)} \in \mathbb{C}$ denote the channel coefficients from $\mathcal{S}_k$ to $\mathcal{R}_i$ and from $\mathcal{R}_i$ to $\mathcal{D}_k$, corresponding to the first and second hops, respectively. The term $h_{in}^{(r)} \in \mathbb{C}$ indicates the channel coefficient between two relays $\mathcal{R}_i$ and $\mathcal{R}_n$. All the channels are assumed to be Rayleigh, having zero-mean and unit variance, and to be independent across different i, k, n, and hop index r. We assume the block-fading model, i.e., the channels are constant during one block (e.g., frame), consisting of one scheduling time slot and L data transmission time slots, and changes to a new independent value for every block.

3 ACHIEVABILITY RESULTS

In this section, we describe the OND protocol, operating in virtual full-duplex mode, in the $K \times N \times K$ channel with interfering relays. Then, its performance is analyzed in terms of achievable DoF along with a certain relay scaling condition. The decaying rate of the interference level is also analyzed. In addition, the OND protocol with no alternate relaying and its achievability result are shown for comparison.

3.1 OND in the $K \times N \times K$ Channel With Interfering Relays

In this subsection, we introduce an OND protocol as the achievable scheme to guarantee the optimal DoF of the $K \times N \times K$ channel with inter-relay interference, where $2K$ relay nodes among N candidates are opportunistically selected for data forwarding in the sense of producing a sufficiently small amount of interference level. The proposed scheme is basically performed by utilizing the channel reciprocity of TDD systems.

Suppose that $\pi_1(k)$ and $\pi_2(k)$ denote the indices of two relays communicating with the kth S–D pair for $k \in \{1, \cdots, K\}$. In this case, without loss of generality, assuming that the number of data transmission time slots, L, is an odd number, the specific steps of each node during one block are described as follows:

- Time slot 1: Sources $\mathcal{S}_1, \cdots, \mathcal{S}_K$ transmit their first encoded symbols $x_1^{(1)}(1), \cdots, x_K^{(1)}(1)$, where $x_k^{(1)}(l)$ represents the lth transmit symbol of the kth source node.[3] A set of K selected relay nodes, $\mathbf{\Pi}_1 = \{\pi_1(1), \cdots, \pi_1(K)\}$, operating in receive mode at each odd time slot, listens to $x_1^{(1)}(1), \cdots, x_K^{(1)}(1)$ (note that a relay selection strategy will be specified later). Other $N - K$ relay nodes and destinations $\mathcal{D}_1, \cdots, \mathcal{D}_K$ remain idle.
- Time slot 2: The K sources transmit their encoded symbols $x_1^{(1)}(2), \cdots, x_K^{(1)}(2)$. The K selected relays in the set $\mathbf{\Pi}_1$ forward their first re-encoded symbols $x_{\pi_1(1)}^{(2)}(1), \cdots, x_{\pi_1(K)}^{(2)}(1)$ to the corresponding K destinations. If the relays in $\mathbf{\Pi}_1$ successfully decode the corresponding symbols, then $x_{\pi_1(k)}^{(2)}(1)$ is the same as $x_k^{(1)}(1)$. Another set of K selected relay nodes, $\mathbf{\Pi}_2 = \{\pi_2(1), \cdots, \pi_2(K)\}$, operating in receive mode at each even time slot, listens to and decodes $x_1^{(1)}(2), \cdots, x_K^{(1)}(2)$ while being interfered with by $\mathcal{R}_{\pi_1(1)}, \cdots, \mathcal{R}_{\pi_1(K)}$. The K destinations receive and decode $x_{\pi_1(1)}^{(2)}(1), \cdots, x_{\pi_1(K)}^{(2)}(1)$ from $\mathcal{R}_{\pi_1(1)}, \cdots, \mathcal{R}_{\pi_1(K)}$. The remaining $N - 2K$ relays keep idle.
- Time slot 3: The K sources transmit their encoded symbols $x_1^{(1)}(3), \cdots, x_K^{(1)}(3)$. The K relays $\pi_2(1), \cdots, \pi_2(K)$ forward their re-encoded symbols $x_{\pi_2(1)}^{(2)}(2), \cdots, x_{\pi_2(K)}^{(2)}(2)$ to the corresponding K destinations. Another K relays in $\mathbf{\Pi}_1$ receive and decode $x_1^{(2)}(3), \cdots, x_K^{(2)}(3)$ while being interfered with by $\mathcal{R}_{\pi_2(1)}, \cdots, \mathcal{R}_{\pi_2(K)}$. The K destinations receive

and decode $x_{\pi_2(1)}^{(2)}(2), \cdots, x_{\pi_2(K)}^{(2)}(2)$ from $\mathcal{R}_{\pi_2(1)}, \cdots, \mathcal{R}_{\pi_2(K)}$. The remaining $N - 2K$ relays keep idle.
- The processes in time slots 2 and 3 are repeated to the $(L-1)$th time slot.
- Time slot L: The K relays in $\mathbf{\Pi}_2$ forward their re-encoded symbols $x_{\pi_2(1)}^{(2)}(L-1), \cdots, x_{\pi_2(K)}^{(2)}(L-1)$ to the corresponding K destinations. The K sources and the other $N - K$ relays remain idle.

At each odd time slot l (i.e., $l = 1, 3, \cdots, L$), let us consider the received signal at each selected relay for the first hop and the received signal at each destination for the second hop, respectively.

For the first hop (Phase 1), the received signal $y_{\pi_1(i)}^{(1)}(l) \in \mathbb{C}$ at $\mathcal{R}_{\pi_1(i)}$ is given by

$$y_{\pi_1(i)}^{(1)}(l) = \sum_{k=1}^{K} h_{\pi_1(i)k}^{(1)} x_k^{(1)}(l) + \sum_{n=1}^{K} h_{\pi_1(i)\pi_2(n)}^{(r)} x_{\pi_2(n)}^{(2)}(l-1)$$
$$+ z_{\pi_1(i)}^{(1)}(l), \qquad (1)$$

where $x_k^{(1)}(l)$ and $x_{\pi_2(n)}^{(2)}(l-1)$ are the lth transmit symbol of $\mathcal{S}_k$ and the $(l-1)$th transmit symbol of $\mathcal{R}_{\pi_2(n)}$, respectively. As addressed earlier, if relay $\mathcal{R}_{\pi_2(k)}$ successfully decodes the received symbol, then it follows that $x_{\pi_2(k)}^{(2)}(l-1) = x_k^{(1)}(l-1)$. The received signal $y_{\pi_1(i)}^{(1)}(l)$ at $\mathcal{R}_{\pi_1(i)}$ is corrupted by the independent and identically distributed (i.i.d.) and circularly symmetric complex additive white Gaussian noise (AWGN) $z_{\pi_1(i)}^{(1)}(l)$ having zero-mean and variance N_0. Note that the second term in the right-hand side (RHS) of (1) indicates the inter-relay interference, which occurs when the K relays in the set $\mathbf{\Pi}_1$, operating in receive mode, listen to the sources, the relays are interfered with by the other set $\mathbf{\Pi}_2$, operating in transmit mode. Note that when $l = 1$, relays have no symbols to transmit, and the second term in the RHS of (1) becomes zero. Similarly when $l = L$, sources do not transmit symbols, and the first term in the RHS of (1) becomes zero.

For the second hop (Phase 2), assuming that the K selected relay nodes transmit their data packets simultaneously, the received signal $y_k^{(2)}(l) \in \mathbb{C}$ at $\mathcal{D}_k$ is given by

$$y_k^{(2)}(l) = \sum_{n=1}^{K} h_{k\pi_2(n)}^{(2)} x_{\pi_2(n)}^{(2)}(l-1) + z_k^{(2)}(l), \qquad (2)$$

where $z_k^{(2)}(l)$ is the i.i.d. AWGN having zero-mean and variance N_0. We also note that when $l = 1$, there are no signals from relays.

Likewise, at each even time slot (i.e., $l = 2, 4, \cdots, L - 1$), the received signals at $\mathcal{R}_{\pi_2(i)}$ and $\mathcal{D}_k$ (i.e., the first and

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3. For notational convenience, we use scalar notation instead of vector notation for each coding block from source nodes, but the size of each symbol is assumed to be sufficiently long to achieve the Shannon-theoretic channel capacity.

second hops) are given by

$$y^{(1)}_{\pi_2(i)}(l) = \sum_{k=1}^{K} h^{(1)}_{\pi_2(i)k} x^{(1)}_k(l) + \sum_{n=1}^{K} h^{(r)}_{\pi_2(i)\pi_1(n)} x^{(2)}_{\pi_1(n)}(l-1) + z^{(1)}_{\pi_2(i)}(l)$$

and

$$y^{(2)}_k(l) = \sum_{n=1}^{K} h^{(2)}_{k\pi_1(n)} x^{(2)}_{\pi_1(n)}(l-1) + z^{(2)}_k(l),$$

respectively. The illustration of the aforementioned OND protocol is geographically shown in Fig. 1 (two terms $\tilde{L}_{\pi_1(k),k}$ and $L^{\Pi_2}_{\pi_2(k),k}$ are specified later in the following relay selection steps).

Now, let us describe how to choose two types of relay sets, Π_1 and Π_2, among N relay nodes, where N is sufficiently large (the minimum N required to guarantee the DoF optimality will be analyzed in Section 3.2).

3.1.1 Step 1 (The First Relay Set Selection)

Let us first focus on selecting the set $\Pi_1 = \{\mathcal{R}_{\pi_1(1)}, \cdots, \mathcal{R}_{\pi_1(K)}\}$, operating in receive and transmit modes in odd and even time slots, respectively. For every scheduling period, it is possible for relay $\mathcal{R}_i$ to obtain all the channel coefficients $h^{(1)}_{ik}$ and $h^{(2)}_{ki}$ by using a pilot signaling sent from all of the source and destination nodes due to the channel reciprocity before data transmission, where $i \in \{1, \cdots, N\}$ and $k \in \{1, \cdots, K\}$ (note that this is our local CSI assumption). When $\mathcal{R}_i$ is assumed to serve the kth S–D pair $(\mathcal{S}_k, \mathcal{D}_k)$, it then examines both i) how much interference is received from the other sources and ii) how much interference is generated by itself to the other destinations, by computing the following scheduling metric $\tilde{L}_{i,k}$:

$$\tilde{L}_{i,k} = \sum_{\substack{m=1 \\ m \neq k}}^{K} \left(\left| h^{(1)}_{im} \right|^2 + \left| h^{(2)}_{mi} \right|^2 \right), \quad (3)$$

where $i \in \{1, \ldots, N\}$ and $k \in \{1, \ldots, K\}$. We remark that the first term $\sum_{m=1, m \neq k}^{K} \left| h^{(1)}_{im} \right|^2$ in (3) denotes the sum of interference power received at $\mathcal{R}_i$ for the first hop (i.e., Phase 1). On the other hand, the second term $\sum_{m=1, m \neq k}^{K} \left| h^{(2)}_{mi} \right|^2$ indicates the sum of interference power generating at $\mathcal{R}_i$, which can be interpreted as the *leakage of interference* to the $K-1$ receivers expect for the corresponding destination, for the second hop (i.e., Phase 2) under the same assumption.

Suppose that a short duration CTS (Clear to Send) message is transmitted by the destination who finds its desired relay node (or the master destination). Then according to the computed metrics $\tilde{L}_{i,k}$ in (3), a timer-based method can

be used for the relay selection similarly as in [31].[4] Note that the method based on the timer is considerably suitable in distributed systems in the sense that information exchange among all the relay nodes can be minimized. At the beginning of every scheduling period, the relay $\mathcal{R}_i$ computes the set of K scheduling metrics, $\{\tilde{L}_{i,1}, \cdots, \tilde{L}_{i,K}\}$, and then starts its own timer with K initial values, which can be set to be proportional to the K metrics.[5] Thus, there exist NK metrics over the whole relay nodes, and we need to compare them so as to determine who will be selected. The timer of the relay $\mathcal{R}_{\pi_1(\hat{k})}$ with the least one $\tilde{L}_{\pi_1(\hat{k}),\hat{k}}$ among NK metrics will expire first, where $\pi_1(\hat{k}) \in \{1, \cdots, N\}$ and $\hat{k} \in \{1, \cdots, K\}$. The relay then transmits a short duration RTS message, signaling its presence, to the other $N-1$ relays, where each RTS message is composed of $\lceil \log_2 K \rceil$ bits to indicate which S–D pair the relay wants to serve. Thereafter, the relay $\mathcal{R}_{\pi_1(\hat{k})}$ is first selected to forward the $\hat{k}$th S–D pair's packet. All the other relays are in listen mode while waiting for their timer to be set to zero (i.e., to expire). At the stage of deciding who will send the second RTS message, it is assumed that the other relays are not allowed to communicate with the $\hat{k}$th S–D pair, and thus the associated metrics $\{\tilde{L}_{1,\hat{k}}, \cdots, \tilde{L}_{\pi_1(\hat{k})-1,\hat{k}}, \tilde{L}_{\pi_1(\hat{k})+1,\hat{k}}, \cdots, \tilde{L}_{N,\hat{k}}\}$ are discarded with respect to timer operation. If another relay has an opportunity to send the second RTS message of $\lceil \log_2(K-1) \rceil$ bits in order to declare its presence, then it is selected to communicate with the corresponding S–D pair. When such K RTS messages, consisting of at most $K \lceil \log_2 K \rceil$ bits, are sent out in consecutive order, i.e., the set of K relays, $\Pi_1 = \{\mathcal{R}_{\pi_1(1)}, \cdots, \mathcal{R}_{\pi_1(K)}\}$, is chosen, the timer-based algorithm for the first relay set selection terminates, yielding no RTS collision with high probability. We remark that when $K = 1$ (i.e., the single S–D pair case), K relay nodes are *arbitrarily* chosen as the first relay set Π_1 since there is no interference in this step.

3.1.2 Step 2 (The Second Relay Set Selection)

Now let us turn to choosing the set of K relay nodes (among $N-K$ candidates), $\Pi_2 = \{\pi_2(1), \cdots, \pi_2(K)\}$, operating in receive and transmit modes in even and odd

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4. The reception of a CTS message, which is transmitted from a certain destination, triggers the initial timing process at each relay. Therefore, no explicit timing synchronization protocol is required among the relays [31], [32]. Moreover, it is worth noting that the overhead of relay selection is a small fraction of one transmission block with small collision probability [31]. Since our relay selection procedure is performed sequentially over all the S-D pairs and the already selected relays for a certain S-D pair are not allowed to take part in the selection process for another S-D pair, the collision probability is thus at most $2K$ times that of the single S-D pair case [31].

5. To avoid a situation such that a malicious relay deliberately sets its timer to a smaller value so as to win the chance, prior to the relay selection process, a secret key may be shared among legitimate nodes including relays. If a malicious relay who did not share the key wants to participate in communication, then one can neglect his/her message (e.g., RTS (Request to Send) message).

time slots, respectively. Using K RTS messages broadcasted from the K relay nodes in the set $\mathbf{\Pi}_1$, it is possible for relay node $\mathcal{R}_i \in \{1, \cdots, N\} \setminus \mathbf{\Pi_1}$ to compute the sum of inter-relay interference power generated from the relays in $\mathbf{\Pi}_1$, denoted by $\sum_{k=1}^{K} \left| h_{i\pi_1(k)}^{(r)} \right|^2$. When $\mathcal{R}_i$ is again assumed to serve the kth S–D pair $(\mathcal{S}_k, \mathcal{D}_k)$, it examines both i) how much interference is received from the undesired sources and the selected relays in the set $\mathbf{\Pi}_1$ for the first hop and ii) how much interference is generated by itself to the other destinations by computing the following metric $L_{i,k}^{\mathbf{\Pi_2}}$, termed *total interference level (TIL)*:

$$
\begin{aligned}
L_{i,k}^{\mathbf{\Pi_2}} &= \tilde{L}_{i,k} + \sum_{k=1}^{K} \left| h_{i\pi_1(k)}^{(r)} \right|^2 \\
&= \sum_{\substack{m=1 \\ m \neq k}}^{K} \left(\left| h_{im}^{(1)} \right|^2 + \left| h_{mi}^{(2)} \right|^2 \right) + \sum_{k=1}^{K} \left| h_{i\pi_1(k)}^{(r)} \right|^2, \quad (4)
\end{aligned}
$$

where $i \in \{1, \ldots, N\}$ and $k \in \{1, \ldots, K\}$. We note that Steps 1 and 2 cannot be exchangeable due to the fact that the inter-relay interference term $\sum_{k=1}^{K} \left| h_{i\pi_1(k)}^{(r)} \right|^2$ is measured after determining the first relay set $\mathbf{\Pi}_1$. If the relay set selection order is switched, then the metric TIL in (4) will not be available.

According to the computed TIL $L_{i,k}^{\mathbf{\Pi_2}}$, we also apply the timer-based method used in Step 1 for the second relay set selection. The relay $\mathcal{R}_i \in \{1, \cdots, N\} \setminus \mathbf{\Pi_1}$ computes the set of K TILs, $\{L_{i,1}^{\mathbf{\Pi_2}}, \cdots, L_{i,K}^{\mathbf{\Pi_2}}\}$, and then starts its timer with K initial values, proportional to the K TILs. Thus, we need to compare $(N-K)K$ TIL metrics over the relay nodes in the set $\{1, \cdots, N\} \setminus \mathbf{\Pi_1}$ in order to determine who will be selected as the second relay set. The rest of the relay set selection protocol (i.e., RTS message exchange among relay nodes) almost follows the same line as that of Step 1. The timer-based algorithm for the second relay set selection terminates when K RTS messages are sent out in consecutive order. Then, K relay nodes having a sufficiently small amount of TIL $L_{i,k}^{\mathbf{\Pi_2}}$ are selected as the second relay set $\mathbf{\Pi}_2$.

Remark 1. Owing to the channel reciprocity of TDD systems, the sum of inter-relay interference power received at any relay $R_i \in \mathbf{\Pi_1}$, $\sum_{k=1}^{K} \left| h_{i\pi_2(k)}^{(r)} \right|^2$, also turns out to be sufficiently small when N is large. That is, it is also guaranteed that K selected relays in the set $\mathbf{\Pi}_1$ have a sufficiently small amount of TIL.

Remark 2. The overhead of each scheduling time slot (i.e., the total number of bits required for exchanging RTS messages among the relay nodes) can be made arbitrarily small, compared to one transmission block. From the fact that K RTS messages, consisting of at most $K \lceil \log_2 K \rceil$

bits, are sent out in each relay set selection step, only $2K \lceil \log_2 K \rceil$ bit transmission could suffice.

3.1.3 Step 3 (Data Transmission)

The $2K$ selected relays request data transmission to their desired source nodes. Each source ($\mathcal{S}_k$) then starts to transmit data to the corresponding destination ($\mathcal{D}_k$) via one of its two relay nodes alternately ($\mathcal{R}_{\pi_1(k)}$ or $\mathcal{R}_{\pi_2(k)}$), which was specified earlier. If the TILs of the selected relays are arbitrarily small, then i) the associated undesired source–relay and relay–destination channel links and ii) the inter-relay channel links are all in deep fade. In Section 3.2, we will show that it is possible to choose such relays with the help of the multiuser diversity gain.

At the receiver side, each relay or destination detects the signal sent from its desired transmitter, while simply treating interference as Gaussian noise. Thus, no multiuser detection is performed at each receiver, thereby resulting in an easier implementation.

3.2 Analysis of a Lower Bound on the DoF

In this subsection, using the scaling argument bridging between the number of relays, N, and the received SNR (refer to [9], [18], [19], [20] for the details), we shall show 1) the lower bound on the DoF of the $K \times N \times K$ channel with interfering relays as N increases and 2) the minimum N required to guarantee the achievability result. The total number of DoF, denoted by $\mathsf{DoF}_{\text{total}}$, is defined as [2]

$$
\mathsf{DoF}_{\text{total}} = \sum_{k=1}^{K} \left(\lim_{\mathsf{snr} \to \infty} \frac{T_k(\mathsf{snr})}{\log \mathsf{snr}} \right),
$$

where $T_k(\mathsf{snr})$ denotes the transmission rate of source $\mathcal{S}_k$. Using the OND framework in the $K \times N \times K$ channel with interfering relays where L transmission slots per block are used, the achievable $\mathsf{DoF}_{\text{total}}$ is lower-bounded by

$$
\mathsf{DoF}_{\text{total}} \geq \frac{L-1}{L} \sum_{k=1}^{K} \sum_{s=1}^{2}
$$

$$
\left(\lim_{\mathsf{snr} \to \infty} \frac{\frac{1}{2} \log \left(1 + \min \left\{ \mathsf{sinr}_{\pi_s(k)}^{(1)}, \mathsf{sinr}_{k,\pi_s(k)}^{(2)} \right\} \right)}{\log \mathsf{snr}} \right), \quad (5)
$$

where $\mathsf{sinr}_{\pi_s(k)}^{(1)}$ denotes the received signal-to-interference-and-noise ratio (SINR) at the relay $\mathcal{R}_{\pi_s(k)}$ and $\mathsf{sinr}_{k,\pi_s(k)}^{(2)}$ denotes the received SINR at the destination $\mathcal{D}_k$ when the relay $\mathcal{R}_{\pi_s(k)}$ transmits the desired signal ($s = 1, 2$ and $k \in \{1, \cdots, K\}$). More specifically, the above SINRs can

be formally expressed as[6]

$$\text{sinr}_{\pi_s(i)}^{(1)} = \frac{P\left|h_{\pi_s(i)i}^{(1)}\right|^2}{N_0 + P\sum_{\substack{k=1\\k\neq i}}^{K}\left|h_{\pi_s(i)k}^{(1)}\right|^2 + P\sum_{k=1}^{K}\left|h_{\pi_s(i),\pi_{\tilde{s}}(k)}^{(r)}\right|^2}$$

$$\text{sinr}_{i,\pi_s(i)}^{(2)} = \frac{P\left|h_{i\pi_s(i)}^{(2)}\right|^2}{1 + P\sum_{\substack{k=1\\k\neq i}}^{K}\left|h_{i\pi_s(k)}^{(2)}\right|^2},$$

where the second term in the denominator of $\text{sinr}_{\pi_s(i)}^{(1)}$ indicates the interference power at relay $\pi_s(i)$ received from the sources while the third term indicates the inter-relay interference, and the second term in the denominator of $\text{sinr}_{i,\pi_s(i)}^{(2)}$ indicates the interference power at the destination $\mathcal{D}_i$ received from the active relays. Here, $\tilde{s} = 3-s$, i.e., $\tilde{s} = 2$ if $s = 1$, and vice versa.

We focus on the first relay set $\mathbf{\Pi}_1$'s perspective to examine the received SINR values according to each time slot. Let us first denote $L_{\pi_1(i),i}^{\mathbf{\Pi}_1} \triangleq \tilde{L}_{\pi_1(i),i} + \sum_{k=1}^{K}\left|h_{\pi_1(i),\pi_2(k)}^{(r)}\right|^2$ for $i \in \{1, \cdots, K\}$. For the first hop, at time slot $2t-1$ (i.e., each odd time slot), $t \in \left\{1, 2, \ldots, \frac{L-1}{2}\right\}$, the received $\text{sinr}_{\pi_1(i)}^{(1)}$ at $\mathcal{R}_{\pi_1(i)}^{(1)}$ is lower-bounded by

$$\text{sinr}_{\pi_1(i)}^{(1)}$$

$$\geq \frac{\text{snr}\left|h_{\pi_1(i)i}^{(1)}\right|^2}{1 + \text{snr}\sum_{\substack{k=1\\k\neq i}}^{K}\left(\left|h_{\pi_1(i)k}^{(1)}\right|^2 + \left|h_{k\pi_1(i)}^{(2)}\right|^2\right) + \text{snr}\sum_{k=1}^{K}\left|h_{\pi_1(i),\pi_2(k)}^{(r)}\right|^2}$$

$$= \frac{\text{snr}\left|h_{\pi_1(i)i}^{(1)}\right|^2}{1 + \text{snr}\left(\tilde{L}_{\pi_1(i),i} + \sum_{k=1}^{K}\left|h_{\pi_1(i),\pi_2(k)}^{(r)}\right|^2\right)}$$

$$= \frac{\text{snr}\left|h_{\pi_1(i)i}^{(1)}\right|^2}{1 + \text{snr}L_{\pi_1(i),i}^{\mathbf{\Pi}_1}}$$

$$\geq \frac{\text{snr}\left|h_{\pi_1(i)i}^{(1)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}L_{\pi_1(i),i}^{\mathbf{\Pi}_1}}, \tag{6}$$

where $\tilde{L}_{\pi_1(i),i}$ indicates the scheduling metric in (3) when $\mathcal{R}_{\pi_1(i)}$ is assumed to serve the ith S–D pair, $(\mathcal{S}_i, \mathcal{D}_i)$. For the second hop, at time slot $2t$ (i.e., each even time slot), $t \in \left\{1, 2, \ldots, \frac{L-1}{2}\right\}$, the received $\text{sinr}_{i,\pi_1(i)}^{(2)}$ at $\mathcal{D}_i$ is lower-bounded by

$$\text{sinr}_{i,\pi_1(i)}^{(2)} \geq \frac{\text{snr}\left|h_{i\pi_1(i)}^{(2)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}\sum_{\substack{k=1\\k\neq i}}^{K}\left|h_{i\pi_1(k)}^{(2)}\right|^2}$$

$$\geq \frac{\text{snr}\left|h_{i\pi_1(i)}^{(2)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}\sum_{\substack{k=1\\k\neq i}}^{K}\left(\left|h_{\pi_1(i)k}^{(1)}\right|^2 + \left|h_{k\pi_1(i)}^{(2)}\right|^2\right)}$$

$$= \frac{\text{snr}\left|h_{i\pi_1(i)}^{(2)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}\tilde{L}_{\pi_1(i),i}}$$

$$\geq \frac{\text{snr}\left|h_{i\pi_1(i)}^{(2)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}L_{\pi_1(i),i}^{\mathbf{\Pi}_1}}, \tag{7}$$

where the second inequality holds due to the channel reciprocity. The term $\sum_{i=1}^{K}L_{\pi_1(i),i}^{\mathbf{\Pi}_1}$ in the denominator of (6) and (7) needs to scale as snr^{-1}, i.e., $\sum_{i=1}^{K}L_{\pi_1(i),i}^{\mathbf{\Pi}_1} = O(\text{snr}^{-1})$, so that both $\text{sinr}_{\pi_1(k)}^{(1)}$ and $\text{sinr}_{k,\pi_1(k)}^{(2)}$ scale as $\Omega(\text{snr})$ with increasing SNR, which eventually enables to achieve the DoF of $\frac{L-1}{L}$ per S–D pair from (5).[7] Even if such a bounding technique in (6) and (7) leads to a loose lower bound on the SINR, it is sufficient to prove our achievability result in terms of DoF and relay scaling law.

Now, let us turn to the second relay set $\mathbf{\Pi}_2$. Similarly as in (6), for the first hop, at time slot $2t$, $t \in \left\{1, 2, \ldots, \frac{L-1}{2}\right\}$, the received $\text{sinr}_{\pi_2(i)}^{(1)}$ at $\mathcal{R}_{\pi_2(i)}^{(1)}$ is lower-bounded by

$$\text{sinr}_{\pi_2(i)}^{(1)} \geq \frac{\text{snr}\left|h_{\pi_2(i)i}^{(1)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}L_{\pi_2(i),i}^{\mathbf{\Pi}_2}}, \tag{8}$$

where $L_{\pi_2(i),i}^{\mathbf{\Pi}_2}$ indicates the TIL in (4) when $\mathcal{R}_{\pi_2(i)}$ is assumed to serve the ith S–D pair $(\mathcal{S}_i, \mathcal{D}_i)$. For the second hop, at time slot $2t + 1$, $t \in \left\{1, 2, \ldots, \frac{L-1}{2}\right\}$, the received $\text{sinr}_{i,\pi_2(i)}^{(2)}$ at $\mathcal{D}_i$ can also be lower-bounded by

$$\text{sinr}_{i,\pi_2(i)}^{(2)} \geq \frac{\text{snr}\left|h_{i\pi_2(i)}^{(2)}\right|^2}{1 + \text{snr}\sum_{i=1}^{K}L_{\pi_2(i),i}^{\mathbf{\Pi}_2}}. \tag{9}$$

The next step is thus to characterize the three metrics $\tilde{L}_{i,k}$, $L_{i,k}^{\mathbf{\Pi}_1}$, and $L_{i,k}^{\mathbf{\Pi}_2}$ ($i \in \{1, \ldots, N\}$ and $k \in \{1, \ldots, K\}$) and their cumulative density functions (CDFs) in the $K \times N \times K$ channel with interfering relays, which is used to analyze the lower bound on the DoF and the required relay scaling law in the model under consideration. Since it is obvious to show that the CDF of $L_{i,k}^{\mathbf{\Pi}_1}$ is identical to that of $L_{i,k}^{\mathbf{\Pi}_2}$, we focus only on the characterization of $L_{i,k}^{\mathbf{\Pi}_2}$. The scheduling metric $\tilde{L}_{i,k}$ follows the chi-square distribution with $2(2K-2)$ degrees of freedom since it represents the sum of i.i.d. $2K-2$ chi-square random variables with 2 degrees of freedom. Similarly, the TIL $L_{i,k}^{\mathbf{\Pi}_2}$ follows the chi-square distribution with $2(3K-2)$ degrees of freedom. The

---

6. Note that at the first time slot for the relays $\{\pi_1(i)\}_{i=1}^{K}$, the third term in the denominator of $\text{sinr}_{\pi_1(i)}^{(1)}$ (i.e., the inter-relay interference term) becomes zero.

7. We use the following notation: i) $f(x) = O(g(x))$ means that there exist constant C and c such that $f(x) \leq Cg(x)$ for all $x > c$. ii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$. iii) $f(x) = \omega(g(x))$ means that $\lim_{x\to\infty}\frac{g(x)}{f(x)} = 0$ [33].

CDFs of the two metrics $\tilde{L}_{i,k}$ and $L_{i,k}^{\Pi_2}$ are given by

$$\mathcal{F}_{\tilde{L}}(\ell) = \frac{\gamma(2K-2, \ell/2)}{\Gamma(2K-2)} \tag{10}$$

$$\mathcal{F}_{L}(\ell) = \frac{\gamma(3K-2, \ell/2)}{\Gamma(3K-2)}, \tag{11}$$

respectively, where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function and $\gamma(z, x) = \int_0^x t^{z-1} e^{-t} dt$ is the lower incomplete Gamma function [34, eqn. (8.310.1)]. We start from the following lemma.

Lemma 1. For any $0 < \ell \le 2$, the CDFs of the random variables $\tilde{L}_{i,k}$ and $L_{i,k}^{\Pi_2}$ in (10) and (11) are lower-bounded by $\mathcal{F}_{\tilde{L}}(\ell) \ge C_1 \ell^{2K-2}$ and $\mathcal{F}_{L}(\ell) \ge C_2 \ell^{3K-2}$, respectively, where

$$C_1 = \frac{e^{-1} 2^{-(2K-2)}}{\Gamma(2K-1)} \tag{12}$$

$$C_2 = \frac{e^{-1} 2^{-(3K-2)}}{\Gamma(3K-1)}, \tag{13}$$

and $\Gamma(z)$ is the Gamma function.

Proof: The detailed proof of this argument is omitted here since it essentially follows the similar line to the proof of [9, Lemma 1] with a slight modification. □

In the following theorem, we establish our first main result by deriving the lower bound on the total DoF in the $K \times N \times K$ channel with interfering relays.

Theorem 1. Suppose that the OND scheme with alternate relaying is used for the $K \times N \times K$ channel with interfering relays. Then, for L data transmission time slots,

$$\mathsf{DoF}_{\text{total}} \ge \frac{(L-1)K}{L}$$

is achievable if $N = \omega\left(\mathsf{snr}^{3K-2}\right)$.

Proof: From (5)–(9), the OND scheme achieves $\mathsf{DoF}_{\text{total}} \ge \frac{L-1}{L}K$ provided that the two values $\mathsf{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1}$ and $\mathsf{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2}$ are less than or equal to some constant $\epsilon_0 > 0$, independent of SNR, for all S–D pairs. Then, a lower bound on the achievable $\mathsf{DoF}_{\text{total}}$ is given by

$$\mathsf{DoF}_{\text{total}} \ge \mathcal{P}_{\text{OND}} \frac{(L-1)K}{L},$$

which indicates that $\frac{L-1}{L}K$ DoF is achievable for a fraction $\mathcal{P}_{\text{OND}}$ of the time for actual transmission, where

$$\mathcal{P}_{\text{OND}}$$
$$= \lim_{\mathsf{snr} \to \infty} \Pr\left\{ \mathsf{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \le \epsilon_0 \text{ and } \mathsf{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \le \epsilon_0 \right\}. \tag{14}$$

We now examine the relay scaling condition such that $\mathcal{P}_{\text{OND}}$ converges to one with high probability. For the sim-

plicity of the proof, suppose that the first and the second relay sets Π_1 and Π_2 are selected out of two mutually exclusive relaying candidate sets $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively, i.e., $\mathcal{N}_1, \mathcal{N}_2 \subset \{1, \ldots, N\}$, $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$, $\mathcal{N}_1 \cup \mathcal{N}_2 = \{1, \ldots, N\}$, $\Pi_1 \subset \mathcal{N}_1$, and $\Pi_2 \subset \mathcal{N}_2$. Then, we are interested in how $|\mathcal{N}_1|$ and $|\mathcal{N}_2|$ scale with SNR in order to guarantee that $\mathcal{P}_{\text{OND}}$ tends to one, where $|\mathcal{N}_s|$ denotes the cardinality of $\mathcal{N}_s$ for $s = 1, 2$. From (14), we further have

$$\mathcal{P}_{\text{OND}} = \lim_{\mathsf{snr} \to \infty} \left(\Pr\left\{ \mathsf{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \le \epsilon_0 \right\} \right.$$
$$\left. \cdot \Pr\left\{ \mathsf{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \le \epsilon_0 \right\} \right). \tag{15}$$

Let $\mathcal{B}_m \triangleq \mathcal{N}_2 \setminus \{\pi_2(\ell)\}_{\ell=1}^{m-1}$ with $\{\pi_2(\ell)\}_{\ell=1}^0 = \emptyset$ and $|\mathcal{B}_m|$ be the candidate set associated with the second relay set and the mth S-D pair and its cardinality, respectively. For a constant $\epsilon_0 > 0$, we can bound the second term in (15) as follows:

$$\Pr\left\{ \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \le \frac{\epsilon_0}{\mathsf{snr}} \right\}$$
$$\ge \Pr\left\{ \max_{1 \le i \le K} L_{\pi_2(i),i}^{\Pi_2} \le \frac{\epsilon_0}{K\mathsf{snr}} \right\}$$
$$\overset{(a)}{\ge} 1 - \Pr\left\{ \exists i : L_{\pi_2(i),i}^{\Pi_2} \ge \frac{\epsilon_0}{K\mathsf{snr}} \right\}$$
$$\overset{(b)}{\ge} 1 - \sum_{i=1}^K \Pr\left\{ L_{\pi_2(i),i}^{\Pi_2} \ge \frac{\epsilon_0}{K\mathsf{snr}} \right\}$$
$$\overset{(c)}{\ge} 1 - K\Pr\left\{ \min_{j \in \mathcal{N}_2} L_{j,i}^{\Pi_2} \ge \frac{\epsilon_0}{K\mathsf{snr}} \right\}$$
$$\overset{(d)}{\ge} 1 - K\left(1 - \mathcal{F}_L\left(\frac{\epsilon_0}{K\mathsf{snr}}\right)\right)^{|\mathcal{B}_i|}$$
$$\overset{(e)}{\ge} 1 - K\left(1 - C_2\left(\frac{\epsilon_0}{K\mathsf{snr}}\right)^{3K-2}\right)^{|\mathcal{N}_2|-K+1}, \tag{16}$$

where the inequality (a) holds from the De Morgan's law; (b) follows from the union bound; (c) follows since $L_{\pi_2(i),i}^{\Pi_2} = \min_{j \in \mathcal{N}_2} L_{j,i}^{\Pi_2}$; (d) follows since $L_{j,i}^{\Pi_2}$ are the i.i.d. random variables $\forall j \in \mathcal{N}_2$ for a given i, owning to the fact that the channels are i.i.d. variables; and (e) follows from Lemma 1 with $C_2 = \frac{e^{-1} 2^{-(3K-2)}}{\Gamma(3K-1)}$ since $0 < \frac{\epsilon_0}{\mathsf{snr}} \le 2$ as $\mathsf{snr} \to \infty$ and from the fact that $|\mathcal{B}_i| \ge |\mathcal{N}_2| - K + 1$.

We now pay our attention to the first term in (15), which can be bounded by

$$\Pr\left\{ \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \le \frac{\epsilon_0}{\mathsf{snr}} \right\} \ge \Pr\left\{ \max_{1 \le i \le K} L_{\pi_1(i),i}^{\Pi_1} \le \frac{\epsilon_0}{K\mathsf{snr}} \right\}$$
$$= \left(\Pr\left\{ L_{\pi_1(i),i}^{\Pi_1} \le \frac{\epsilon_0}{K\mathsf{snr}} \right\} \right)^K, \tag{17}$$

where the equality follows from the fact that $L_{\pi_1(i),i}^{\Pi_1}$ and $L_{\pi_1(j),j}^{\Pi_1}$ for $i \ne j$ are the functions of different random

variables and thus are independent of each other. By letting $K_i = \sum_{k=1}^{K} \left| h_{\pi_1(i),\pi_2(k)}^{(r)} \right|^2$, by the definition of $L_{\pi_1(i),i}^{\mathbf{\Pi_1}}$, we have

$$\Pr\left\{ L_{\pi_1(i),i}^{\mathbf{\Pi_1}} \leq \frac{\epsilon_0}{K\,\mathsf{snr}} \right\}$$
$$= 1 - \Pr\left\{ \tilde{L}_{\pi_1(i),i} + K_i \geq \frac{\epsilon_0}{K\,\mathsf{snr}} \right\}$$
$$\geq 1 - \Pr\left\{ \tilde{L}_{\pi_1(i),i} \geq \frac{\epsilon_0}{2K\,\mathsf{snr}} \right\} - \Pr\left\{ K_i \geq \frac{\epsilon_0}{2K\,\mathsf{snr}} \right\}, \quad (18)$$

where the inequality follows from the fact that for any random variables X and Y, $\Pr\{X + Y \geq \epsilon\} \leq \Pr\left\{X \geq \frac{\epsilon}{2}\right\} + \Pr\left\{Y \geq \frac{\epsilon}{2}\right\}$ [37]. In the same manner, let $\mathcal{A}_m \triangleq \mathcal{N}_1 \setminus \{\pi_1(\ell)\}_{\ell=1}^{m-1}$ with $\{\pi_1(\ell)\}_{\ell=1}^{m-1} = \emptyset$ and $|\mathcal{A}_m|$ be the candidate set associated with the first relay set and the mth S–D pair and its cardinality, respectively. Then, we can bound the first two terms in the RHS of (18) as follows:

$$1 - \Pr\left\{ \tilde{L}_{\pi_1(i),i} \geq \frac{\epsilon_0}{2K\,\mathsf{snr}} \right\}$$
$$= 1 - \Pr\left\{ \min_{j \in \mathcal{N}_1} \tilde{L}_{j,i} \geq \frac{\epsilon_0}{2K\,\mathsf{snr}} \right\}$$
$$= 1 - \left(1 - \mathcal{F}_{\tilde{L}}\left(\frac{\epsilon_0}{2K\,\mathsf{snr}}\right)\right)^{|\mathcal{A}_i|}$$
$$\geq 1 - \left(1 - C_1\left(\frac{\epsilon_0}{2K\,\mathsf{snr}}\right)^{2K-2}\right)^{|\mathcal{N}_1|-K+1}, \quad (19)$$

where the last inequality follows from Lemma 1 with $C_1 = \frac{e^{-1} 2^{-(2K-2)}}{\Gamma(2K-1)}$. Finally, from (16), it follows that $\Pr\left\{ K_i \geq \frac{\epsilon_0}{2K\,\mathsf{snr}} \right\}$ tends to zero as $|\mathcal{N}_2|$ grows large by noting that $L_{\pi_2(i),i}^{\mathbf{\Pi_2}} = \tilde{L}_{\pi_2(i),i} + K_i$ due to the reciprocal property of TDD systems. From (16), (18), and (19), it is obvious that if $|\mathcal{N}_1|$ and $|\mathcal{N}_2|$ scale faster than snr^{2K-2} and snr^{3K-2}, respectively, then

$$\lim_{\mathsf{snr} \to \infty} \left(1 - C_1\left(\frac{\epsilon_0}{2K\,\mathsf{snr}}\right)^{2K-2}\right)^{|\mathcal{N}_1|-K+1} = 0 \quad (20)$$

$$\lim_{\mathsf{snr} \to \infty} \left(1 - C_2\left(\frac{\epsilon_0}{K\,\mathsf{snr}}\right)^{3K-2}\right)^{|\mathcal{N}_2|-K+1} = 0. \quad (21)$$

Therefore, $\mathcal{P}_{\mathrm{OND}}$ asymptotically approaches one, which means that the DoF of $\frac{(L-1)K}{L}$ is achievable with high probability if $N = |\mathcal{N}_1| + |\mathcal{N}_2| = \omega\left(\mathsf{snr}^{3K-2}\right)$. This completes the proof of the theorem. □

Note that the lower bound on the DoF asymptotically approaches K for large L, which implies that our system operates in virtual full-duplex mode. The parameter N required to obtain full DoF (i.e., K DoF) needs to increase exponentially with the number of S–D pairs, K, in order to make the sum of $3K - 2$ interference terms in the TIL metric (4) non-increase with increasing SNR at each relay.[8] Here, from the perspective of each relay in $\mathbf{\Pi_2}$, the SNR exponent $3K - 2$ indicates the total number of interference

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8. Since snr^{2K-2} scales slower than snr^{3K-2}, it does not affect the performance in terms of DoF and relay scaling laws.

links and stems from the following three factors: the sum of interference power received from other sources, the sum of interference power generated to other destinations, and the sum of inter-relay interference power generated from the relays in $\mathbf{\Pi_1}$. From Theorem 1, let us provide the following interesting discussions regarding the DoF achievability.

Remark 3. K DoF can be achieved by using the proposed OND scheme in the $K \times N \times K$ channel with interfering links among relay nodes, if the number of relay nodes, N, scales faster than snr^{3K-2} and the number of transmission slots in one block, L, is sufficiently large. In this case, all the interference signals are almost nulled out at each selected relay by exploiting the multiuser diversity gain. In other words, by applying the OND scheme to the interference-limited $K \times N \times K$ channel such that the channel links are inherently coupled with each other, the links among each S–D path via one relay can be completely decoupled, thus enabling us to achieve the same DoF as in the interference-free channel case.

Remark 4. It is not difficult to show that the centralized relay selection method that maximizes the received SINR (at either the relay or the destination) using global CSI at the transmitter, which is a combinatorial problem with exponential complexity, gives the same relay scaling result $N = \omega\left(\mathsf{snr}^{3K-2}\right)$ along with full DoF. However, even with our OND scheme using a decentralized relay selection based only on local CSI, the same achievability result is obtained, thus resulting in a much easier implementation.

3.3 The TIL Decaying Rate

In this subsection, we analyze the TIL decaying rate under the OND scheme with alternate relaying, which is meaningful since the desired relay scaling law is closely related to the TIL decaying rate with respect to N for given SNR.

Let $L_{K\text{th-min}}$ denote the Kth smallest TIL among the ones that N selected relay nodes compute. Since the K relays yielding the TIL values up to the Kth smallest one are selected, the Kth smallest TIL is the largest among the TILs that the selected relays compute. Similarly as in [35], by Markov's inequality, a lower bound on the average decaying rate of $L_{K\text{th-min}}$ with respect to N, $\mathbb{E}\left[\frac{1}{L_{K\text{th-min}}}\right]$, is given by

$$\mathbb{E}\left[\frac{1}{L_{K\text{th-min}}}\right] \geq \frac{1}{\epsilon} \Pr(L_{K\text{th-min}} \leq \epsilon), \quad (22)$$

where the inequality always holds for $\epsilon > 0$. We denote $\mathcal{P}_K(\epsilon)$ as the probability that there are only K relays satisfying TIL $\leq \epsilon$, which is expressed as

$$\mathcal{P}_K(\epsilon) = \binom{N}{K} \mathcal{F}_L(\epsilon)^K (1 - \mathcal{F}_L(\epsilon))^{N-K}, \quad (23)$$

where $\mathcal{F}_L(\epsilon)$ is the CDF of the TIL. Since $\Pr(L_{K\text{th-min}} \leq \epsilon)$ is lower-bounded by $\Pr(L_{K\text{th-min}} \leq \epsilon) \geq \mathcal{P}_K(\epsilon)$, a lower bound on the average TIL decaying rate is given by

$$\mathbb{E}\left[\frac{1}{L_{K\text{th-min}}}\right] \geq \frac{1}{\epsilon}\mathcal{P}_K(\epsilon). \tag{24}$$

The next step is to find the parameter $\hat{\epsilon}$ that maximizes $\mathcal{P}_K(\epsilon)$ in terms of ϵ in order to provide the tightest lower bound.

Lemma 2. When a constant $\hat{\epsilon}$ satisfies the condition $\mathcal{F}_L(\hat{\epsilon}) = K/N$, $\mathcal{P}_K(\hat{\epsilon})$ in (23) is maximized for a given N.

Proof: To find the parameter $\hat{\epsilon}$ that maximizes $\mathcal{P}_K(\epsilon)$, we take the first derivative with respect to ϵ, resulting in

$$\frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon}$$
$$= \frac{\partial \mathcal{F}_T(\epsilon)}{\partial \epsilon}\binom{N}{K}\mathcal{F}_T(\epsilon)^{K-1}(1 - \mathcal{F}_T(\epsilon))^{N-K-1}(K - N\mathcal{F}_T(\epsilon)),$$

which is zero when

$$\hat{\epsilon} = \mathcal{F}_T^{-1}\left(\frac{K}{N}\right). \tag{25}$$

The parameter $\hat{\epsilon}$ is the unique value that maximizes $\mathcal{P}_K(\epsilon)$ since

$$\left.\frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon}\right|_{\epsilon} > 0 \text{ if } 0 < \epsilon < \hat{\epsilon}$$
$$\left.\frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon}\right|_{\epsilon} > 0 \text{ if } \epsilon \geq \hat{\epsilon},$$

which completes the proof of the lemma.   □

Now, we establish our second main theorem, which shows a lower bound on the TIL decaying rate with respect to N.

Theorem 2. Suppose that the OND scheme with alternate relaying is used for the $K \times N \times K$ channel with interfering relays. Then, the decaying rate of TIL is lower-bounded by

$$\mathbb{E}\left[\frac{1}{L_{K\text{th-min}}}\right] \geq \Theta\left(N^{\frac{1}{3K-2}}\right). \tag{26}$$

Proof: As shown in (24), the TIL decaying rate is lower-bounded by the maximum of $\frac{1}{\epsilon}\mathcal{P}_K(\epsilon)$ over ϵ. By Lemma 2, $\mathcal{P}(\hat{\epsilon})$ is maximized when $\hat{\epsilon} = \mathcal{F}_L^{-1}\left(\frac{K}{N}\right)$. Thus, we have

$$\mathbb{E}\left[\frac{1}{L_{K\text{th-min}}}\right] \geq \frac{1}{\mathcal{F}_L^{-1}(K/N)}\binom{N}{K}\left(\frac{K}{N}\right)^K\left(1 - \frac{K}{N}\right)^{N-K}$$
$$\geq \frac{1}{\mathcal{F}_L^{-1}(K/N)}\left(\frac{N-K+1}{N}\right)^K\left(1 - \frac{K}{N}\right)^{N-K}$$
$$\geq \frac{1}{\mathcal{F}_L^{-1}(K/N)}\left(\frac{1}{K}\right)^K e^{-K}$$
$$\geq \Theta\left(N^{\frac{1}{3K-2}}\right),$$

where the second and third inequalities hold since $\binom{N}{K} \geq \left(\frac{N-K+1}{K}\right)^K$ and $\left(1 - \frac{K}{N}\right)^{N-K} \geq \left(1 - \frac{K}{N}\right)^N \geq e^{-K}$, respectively. By Lemma 1, it follows that $\frac{1}{\mathcal{F}_L^{-1}(K/N)} \geq \left(\frac{C_1 N}{K}\right)^{\frac{1}{3K-2}}$, where C_1 is given by (12). Hence, the last inequality also holds, which completes the proof of the theorem.   □

From Theorem 2, the following valuable insight is provided: the smaller SNR exponent of the relay scaling law, the faster TIL decaying rate with respect to N. This asymptotic result will be verified in a finite N regime via numerical evaluation in Section 5.

3.4 OND Without Alternate Relaying

For comparison, the OND scheme without alternate relaying is also explained in this subsection.

It is worth noting that there exists a trade-off between the lower bound on the DoF and the minimum number of relays required to guarantee our achievability result by additionally introducing the OND protocol without alternate relaying. In the scheme, the first relay set $\mathbf{\Pi}_1$ only participates in data forwarding. That is, the second relay set $\mathbf{\Pi}_2$ does not need to be selected for the OND protocol without alternate relaying. Specifically, the steps of each node during one block are then described as follows:

- Time slot 1: Sources $\mathcal{S}_1, \cdots, \mathcal{S}_K$ transmit their first encoded symbols $x_1^{(1)}(1), \cdots, x_K^{(1)}(1)$, where $x_k^{(1)}(l)$ represents the lth transmitted symbol of the kth source node. A set of K selected relay nodes, $\mathbf{\Pi}_1 = \{\pi_1(1), \cdots, \pi_1(K)\}$, operating in receive mode at each odd time slot, listens to $x_1^{(1)}(1), \cdots, x_K^{(1)}(1)$. Other $N - K$ relay nodes and destinations $\mathcal{D}_1, \cdots, \mathcal{D}_K$ remain idle.

- Time slot 2: The K relays in the set $\mathbf{\Pi}_1$ forward their first re-encoded symbols $x_{\pi_1(1)}^{(2)}(1), \cdots, x_{\pi_1(K)}^{(2)}(1)$ to the corresponding K destinations. The K destinations receive from $\mathcal{R}_{\pi_1(1)}, \cdots, \mathcal{R}_{\pi_1(K)}$ and decode $x_{\pi_1(1)}^{(2)}(1), \cdots, x_{\pi_1(K)}^{(2)}(1)$. The remaining $N - K$ relays keep idle.

- The processes in time slots 1 and 2 are repeated to the $(L-1)$th time slot.

- Time slot L: The K relays in $\mathbf{\Pi}_1$ forward their re-encoded symbols $x_{\pi_1(1)}^{(2)}(L-1), \cdots, x_{\pi_1(K)}^{(2)}(L-1)$ to the corresponding K destinations. The K sources and the other $N - K$ relays remain idle.

When $\mathcal{R}_i$ is assumed to serve the kth S–D pair $(\mathcal{S}_k, \mathcal{D}_k)$ ($i \in \{1, \cdots, N\}$ and $k \in \{1, \cdots, K\}$), it computes the scheduling metric $\tilde{L}_{i,k}$ in (3). According to the computed $\tilde{L}_{i,k}$, a timer based method is used for relay selection as in Section 3.1.1. Because there is no inter-relay interference for the OND scheme without alternate relaying, it is expected that the minimum required N to achieve the optimal DoF is

significantly reduced. Our third main theorem is established as follows.

Theorem 3. Suppose that the OND scheme without alternate relaying is used for the $K \times N \times K$ channel. Then, for L data transmission time slots,

$$\mathsf{DoF}_{\text{total}} \geq \frac{K}{2}$$

is achievable if $N = \omega\left(\mathsf{snr}^{2K-2}\right)$.

Proof: The detailed proof of this argument is omitted here since it basically follows the same line as the proof of Theorem 1. □

In Section 5, it will be also seen that in a finite N regime, there exists the case even where the OND without alternate relaying outperforms that of the OND with alternate relaying in terms of achievable sum-rates via computer simulations.

4 UPPER BOUND FOR DoF

In this section, to show the optimality of the proposed OND scheme in the $K \times N \times K$ channel with interfering relays, which consists of K S–D pairs and N relay nodes, we derive an upper bound on the DoF using the cut-set bound [36] as a counterpart of the lower bound on the total DoF in Section 3.2. Suppose that $\tilde{N}$ relay nodes are active, i.e., receive packets and retransmit their re-encoded ones, simultaneously, where $\tilde{N} \in \{1, \cdots, N\}$. This is a generalized version of our transmission since it is not characterized how many relays need to be activated simultaneously to obtain the optimal DoF. We consider the two cuts L_1 and L_2 dividing our network into two parts in a different manner. Let $\mathcal{S}_{L_i}$ and $\mathcal{D}_{L_i}$ denote the sets of sources and destinations, respectively, for the cut L_i in the network ($i = 1, 2$). For the $K \times N \times K$ channel model with interfering relays, we now use the fact that there is no direct path between an S–D pair. Then, it follows that under L_1, K transmit nodes in $\mathcal{S}_{L_1}$ are on the left of the network, while $\tilde{N}$ active relay nodes and K (final) destination nodes in $\mathcal{D}_{L_2}$ are on the right and act as receivers. In this case, we can create the $K \times (\tilde{N} + K)$ multiple-input multiple-output (MIMO) channel between the two sets of nodes separated by the cut. Similarly, the $(\tilde{N} + K) \times K$ MIMO channel are obtained under the cut L_2. It is obvious to show that DoF for the two MIMO channels is upper-bounded by K. Hence, it turns out that even with the half-duplex assumption, our lower bound on the DoF based on the OND with alternate relaying asymptotically approaches this upper bound on the DoF for large L.

Note that this upper bound is generally derived regardless of whether the number of relays, N, tends to infinity or not, whereas the scaling condition $N = \omega\left(\mathsf{snr}^{3K-2}\right)$ is included in the achievability proof.
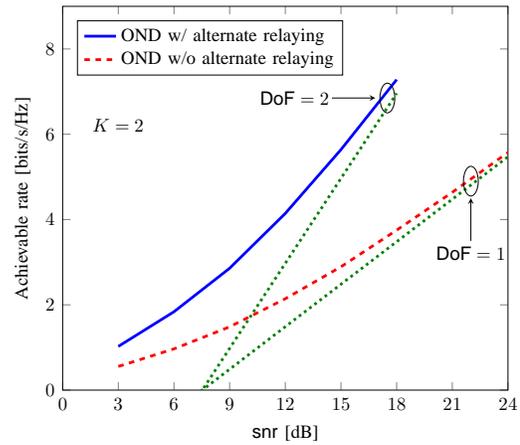

Fig. 2. The achievable sum-rates as a function of snr when $K = 2$ and $N = \mathsf{snr}^4$ in the $K \times N \times K$ channel. It is assumed that $N = \mathsf{snr}^4$ and $N = \mathsf{snr}^2$ are used for the OND schemes with and without alternate relaying, respectively.
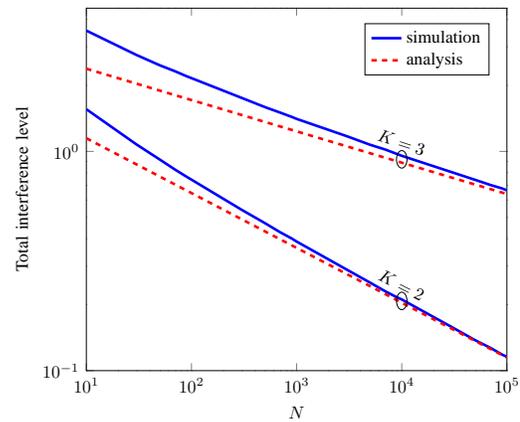

Fig. 3. The average TIL versus N when $K = 2, 3$ in the $K \times N \times K$ channel.

5 NUMERICAL EVALUATION

In this section, we perform computer simulations to validate the achivability result of the proposed OND scheme in Section 3 for finite parameters N and SNR in the $K \times N \times K$ channel model with interfering relays. In our simulation, the channel coefficients in (1) and (2) are generated 1×10^5 times for each system parameter.

Figure 2 shows the achievable sum-rates of the $K \times N \times K$ channel for the OND schemes with and without alternate relaying according to snr in dB scale when $K = 2$. Note that N is set to a different scalable value according to snr, i.e., $N = \mathsf{snr}^{3K-2}$ for the OND with alternate relaying and $N = \mathsf{snr}^{2K-2}$ for the OND without alternate relaying, respectively, to see whether the slope of each curve follows the DoF in Theorems 1 and 2. In the figure, the dotted green lines are also plotted to indicate the first order approximation of the achievable rates with a proper bias, where the slopes are given by K and $K/2$ for the OND schemes with and without alternate relaying, respectively.
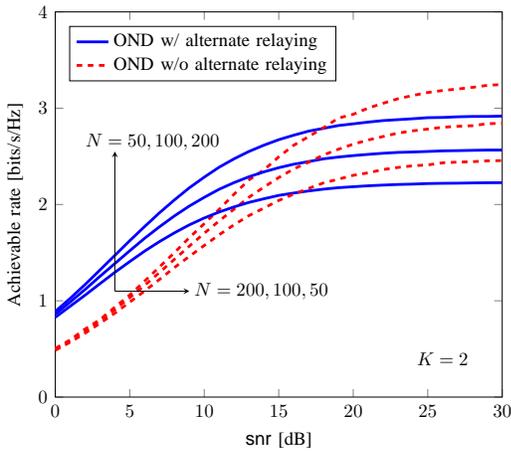
Fig. 4. The achievable sum-rates versus snr when $K = 2$ and $N = 100, 200$ in the $K \times N \times K$ channel. Both OND schemes with and without alternate relaying are compared.
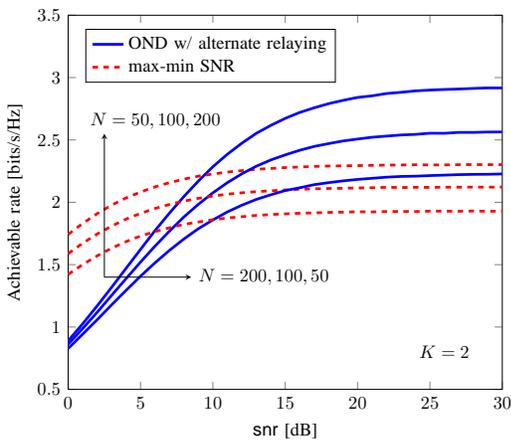

Fig. 5. The achievable sum-rates versus snr when $K = 2$ and $N = 100, 200$ in the $K \times N \times K$ channel. The OND scheme with alternate relaying and the max-min SNR scheme are compared.

In Fig. 3, the log-log plot of the average TIL of the OND with alternate relaying versus N is shown for the $K \times N \times K$ channel when $K = 2, 3$.[9] It can be seen that the TIL tends to decrease linearly with N. It is further seen how many relays are required with the OND scheme with alternate relaying to guarantee that the TIL is less than a small constant for a given parameter K. In this figure, the dashed lines are also plotted from theoretical results in Theorem 2 with a proper bias to check the slope of the TIL. We can see that the TIL decaying rates are consistent with the relay scaling law condition in Theorem 1. More specifically, the TIL is reduced as N increases with the slope of 0.25 for $K = 2$ and 0.143 for $K = 3$, respectively.

Figure 4 illustrates the achievable sum-rates of the $K \times N \times K$ channel for the OND schemes with and without alternate relaying versus snr (in dB scale) when $K = 2$ and

9. Even if it seems unrealistic to have a great number of relays in cooperative relay networks, the range for parameter N is taken into account to precisely see some trends of curves varying with N.

$N = 50, 100, 200$. We can see that in a finite N regime, there exists the case where the OND without alternate relaying outperforms that of the OND with alternate relaying. This is because for finite N, the achievable sum-rates for the alternate relaying case tend to approach a floor with increasing SNR faster than no alternate relaying case due to more residual interference in each dimension. We can also see that the crossing points slightly move to the right as N increases; this is due to the fact that our OND scheme with alternate relaying always benefits from having more relays for selection, thus resulting in more multiuser diversity gain. This highly motivates us to operate our system in a switched fashion when the relay selection scheme is chosen between the OND schemes with and without alternate relaying depending on the operating regime of our system.

To further ascertain the efficacy of our scheme, a performance comparison is performed with a baseline scheduling. Specifically, in the *max-min SNR* scheme, each S–D pair selects one relay $\mathcal{R}_i$ ($i \in \{1, \cdots, N\}$) such that the minimum out of the desired channel gains of two communication links (either from $\mathcal{S}_k$ to $\mathcal{R}_i$ or from $\mathcal{R}_i$ to $\mathcal{D}_k$) becomes the maximum among the associated minimum channel gains over all the unselected relays. This max-min SNR scheme is well-suited for relay-aided systems if interfering links are absent. The achievable sum-rates are illustrated in Fig. 5 according to snr (in dB scale) when $K = 2$ and $N = 100, 200$. We can see that our OND scheme with alternate relaying outperforms this baseline scheme beyond a certain low SNR point. We also see that the rate gaps increase when N increases in the high SNR regime. On the other hand, for fixed N, the sum-rates of the max-min scheme are slightly changed with respect to snr due to more residual interference in each dimension.

6 CONCLUDING REMARKS

An efficient distributed OND protocol operating in virtual full-duplex mode was proposed for the $K \times N \times K$ channel with interfering relays, referred to as one of multi-source interfering relay networks. A novel relay scheduling strategy with alternate half-duplex relaying was presented in two-hop environments, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. It was shown that the OND protocol asymptotically achieves full DoF even in the presence of inter-relay interference and half-duplex assumption, provided that the number of relays, N, scales faster than snr^{3K-2}. Numerical evaluation was also shown to verify that our scheme outperforms the other relay selection methods under realistic network conditions (e.g., finite N and SNR) with respect to sum-rates.

Suggestions for future research in this area include the extension to the MIMO $K \times N \times K$ channel and the optimal design of joint beamforming and scheduling under the MIMO model.

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Won-Yong Shin (S'02–M'08–SM'16) received the B.S. degree in electrical engineering from Yonsei University, Seoul, Korea, in 2002. He received the M.S. and the Ph.D. degrees in electrical engineering and computer science from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2004 and 2008, respectively. From February 2008 to April 2008, he was a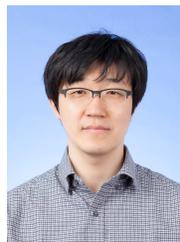 Visiting Scholar in the School of Engineering and Applied Sciences, Harvard University, Cambridge, MA. From September 2008 to April 2009, he was with the Brain Korea Institute and CHiPS at KAIST as a Postdoctoral Fellow. From August 2008 to April 2009, he was with the Lumicomm, Inc., Daejeon, Korea, as a Visiting Researcher. In May 2009, he joined Harvard University as a Postdoctoral Fellow and was promoted to a Research Associate in October 2011. Since March 2012, he has been with the Department of Computer Science and Engineering, Dankook University, Yongin, Korea, where he is currently an Assistant Professor (*early tenured*). His research interests are in the areas of information theory, communications, signal processing, mobile computing, big data analytics, and online social networks analysis.

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