Finite Wordlength Digital Filter Design Using Simulated Annealing

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***Abstract*— This paper describes the design method of a linear phase finite wordlength finite-duration impulse response(FIR) filter using simulated annealing. In many applications, the word length of the FIR filter is limited for various reasons. Main reasons are the computational complexity and the system limitation. Simulated annealing is used to obtain FIR filter coefficients having desired frequency response, since this algorithm has a capability of finding the minimum value of the arbitrary function. Different from previous algorithms, *block update* which changes all the filter coefficients simultaneously and *element-wise update* which change one filter coefficient at once are both considered.**

I. Introduction

Finite-duration impulse response(FIR) filter is widely used in many applications. Besides its simple structure, it has many advantages [1]. In most of the cases, when designing FIR filter, linear phase is considered since linear phase filter have attractive characteristics for digital signal processing and communication applications [2]. Of course, there are needs to design general digital filters [3]. But non-linear phase filter have major effects on the shape of a signal, even when the frequency-response magnitude is constant [2].

If FIR filter is implemented on a specific hardware(e.g. ASIC, FPGA, fixed-point DSP, etc.), wordlength is limited in many cases. Even though there are many design methodologies in designing FIR filter including Remez exchange [4] and linear programming [5], however, these methods does not work in the case of the finite wordlength [6]. Therefore, many design algorithms are developed considering finite wordlength [6]–[9]. These algorithms try to find the optimum coefficients in the sense of Chevyshev or min-max. By defining a cost function which is mean squared error of the desired response and the response of the designed filter in the frequency domain, one can adjust filter coefficients to find the minimum value of the cost function.

The simplest way of designing finite wordlength FIR filter is to round the finite precision solution to the nearest value with the given wordlength. Often integer programming is used [6]. In the integer programming method, filter coefficients are mapped into integer value. Solution can be obtained by normalizing integer coefficients. Branch-and-bound technique can be employed also [7]. Convex property was used to improve efficiency of the branch-and-bound method [10].

Simulated annealing is a method which minimizes function globally regardless of the shape of the function [11], [12]. A minimum of the cost function can be found easily by using this method. This feature can be apply to design FIR filter. In other words, the filter coefficients can be found by minimizing the cost function of the filter response. There are many papers regarding finding coefficients of the FIR filter using simulated annealing [8], [9], [13]. Filter coefficients are searched between $[-1, 1]$. However, searching domain can be break into sub-domains. This sub-domain method leads closer to optimum point and improves speed of simulated annealing. Coefficients update method can be done in two ways : block update and element-wise update. Depending on coefficients update method, convergence speed varies exceedingly.

The remainder of this paper is organized as follows. In section II, a linear phase filter equation is formulated. Next, a modified simulated annealing is presented in section III. A new design methodology is proposed using simulated annealing in section IV. Design example is provided in section V, and section VI concludes the paper.

II. Formulation of the Linear Phase FIR filter

The frequency response of an FIR filter with an N-point impulse response $\{h(k)\}$ is the z-transform of the sequence evaluated on the unit circle [4] :

$$H(f) = H(z)|_{z=e^{j2\pi f}} = \sum_{k=0}^{N-1} h(k)e^{-j2\pi kf} \qquad (1)$$

The frequency response of the linear phase FIR filter can be re-written as

$$H(f) = G(f)e^{j(\frac{L\pi}{2}-(\frac{L-1}{2})2\pi f)} \qquad (2)$$

where $G(f)$is a real valued function and $L = 0$ or $L = 1$. L is determined according to polarity of the filter coefficient. If $\{h(k)\}$ are positive/negative symmetry, $L = 0/1$. Besides, $\{h(k)\}$ can be either odd or even length. Therefore, depending on the polarity and symmetricalness the linear phase FIR filter can be classified into 4 classes [14].

Case 1 : positive symmetry, odd length

$$G(f) = \sum_{k=0}^{n} a(k)\cos(2\pi k f) \tag{3}$$

where $n = (N-1)/2$, $a(0) = h(n)$, and $a(k) = 2h(n-k)$ for $k = 1, 2, \ldots, n$.

Case 2 : positive symmetry, even length

$$G(f) = \sum_{k=1}^{n} b(k)\cos\left[2\pi\left(k - \frac{1}{2}\right)f\right] \tag{4}$$

where $n = N/2$, and $b(k) = 2h(n-k)$ for $k = 1, 2, \ldots, n$.

Case 3 : negative symmetry, odd length

$$G(f) = \sum_{k=1}^{n} c(k)\sin(2\pi k f) \tag{5}$$

where $n = (N-1)/2$, and $c(k) = 2h(n-k)$ for $k = 1, 2, \ldots, n$ and $h(0) = 0$

Case 4 : negative symmetry, even length

$$G(f) = \sum_{k=1}^{n} d(k)\sin\left[2\pi\left(k - \frac{1}{2}\right)f\right] \tag{6}$$

where $n = N/2$, and $d(k) = 2h(n-k)$ for $k = 1, 2, \ldots, n$.

Even though there exists four case of linear phase FIR filters, these representations can be merged into $Case1$ in the form of $G(f) = Q(f)P(f)$ where $P(f)$ is a linear combination of cosine functions [4] [14]. This motivates the single design algorithm. Finite wordlength linear phase FIR filter design problem is to find b-bits(sign included) filter coefficients $\{h(k)\}, n = 1, 2, \ldots, N$. It is equivalent to find frequency response $G(f)$ which minimizes the maximum-weighted absolute error defined as Eq.(7), given desired magnitude response $D(f)$ and positive weight function $W(f)$. In this problem, $D(f)$ and $W(f)$ are both continuous on a compact subset $F \subset [0, \frac{1}{2}]$.

$$\| E(f) \| = \max_{f \in F} W(f)|D(f) - G(f)|. \tag{7}$$

b-bits coefficients $\{h(k)\}, k = 0, 1, \ldots, N-1$ are in the range of $\left[-1, 1 - \frac{1}{2^{b-1}}\right]$ separated by $\frac{1}{2^{b-1}}$. Using symmetry property of $\{h(k)\}$, $G(f)$ can be re-write as follows.

$$G(f) = h(n) + \sum_{k=1}^{n} 2h(n-k)\cos(2\pi k f). \tag{8}$$

where $n = (N-1)/2$

Therefore, using symmetricalness, only n variables instead of N need to be designed according to de desired response.

Minimizing maximum-weighted absolute error is a popular way of defining cost function [6] [14]. However, this criterion does not consider the sidelobe suppression under the desired response at the stop band and may produce high sidelobes. Another criterion to optimize $\{h(k)\}$ is to reduce output error power to minimum. Since this criterion try to minimize overshoot as well as undershoot, it can distort overall frequency response. Therefore, new cost function need to be defined as follows

$$\begin{aligned} E \quad &= \int_{f \in F_{pass}} W(f)|(D(f) - G(f))|^2 df \\ &+ \int_{f \in F_{stop}} W(f)\left|(D(f) - G(f))^+\right|^2 df \end{aligned} \tag{9}$$

where F_{pass} is pass band and F_{stop} is stop band of the filter, $D(f)$ is the given desired magnitude response, $W(f)$ is the positive weight function and $(\cdot)^+$ is the positive part of inside of parentheses.

III. Modified Simulated Annealing

Simulated annealing is the method which is able to find the global extremum(minimum or maximum)[1] of a given bounded cost function [11] [12]. It is based on iterative random search. Since it is not required to calculate derivative, the cost function does not need to be smooth or even continuous in the domain. Furthermore, it may have many local extremum. This feature makes simulated annealing very useful. This method is based on the random walk of each independent coordinates of the cost function. In such way, transitions out of the local minimum are possible. Even though it is not guaranteed to find the global minimum, if the function has many good near-optimal solutions, it should find one [12].

Simulated annealing can start from any initial point x_0 in the domain of interest. Then, the initial value of the cost function ϕ_0 corresponding to the initial value x_0 is calculated. Next, random walk is taken with the step size of Δr producing new point x_1 , therefore the value of the new objective function ϕ_1 can be calculated. Then the difference between ϕ_0 and ϕ_1 is defined as follows.

$$\Delta\phi = \phi_0 - \phi_1 \tag{10}$$

Depending on $\Delta\phi$, new point x_1 is accepted/rejected as a new starting point according to the Metropolis criterion [15] : generate a random number $\rho \in [0, 1]$. And define

$$p = \begin{cases} 1 & \text{if } \Delta\phi \leq 0 \\ \exp\left(-\frac{\Delta\phi}{T}\right) & \text{if } \Delta\phi > 0 \end{cases} \tag{11}$$

where T is a control value called "temperature".

If $\rho \leq p$, x_1 is accepted as a new starting point and becomes x_0. Otherwise, starting point remains at x_0. In Eq. 10, p can be interpreted as a acceptance probability : x_1 is selected as a new starting point with a probability 1 if cost function decreases after random walk. In the case of $\Delta\phi > 0$, x_1 is chosen to be a new starting point with a probability $\exp\left(-\frac{\Delta\phi}{T}\right)$. With the large value of T, it tends to accept x_1 as a new starting point, otherwise it tends to reject. It is recommended that value of T be such that $0.5 < \exp\left(-\frac{\Delta\phi}{T}\right) < 0.9$, because probabilities below 0.5 require too many function evaluations to escape from local minima. And probabilities close to 1 cause inefficient evaluation since it may accept virtually every

[1]Since minimum and maximum problems are different only sign, extremum problem is treated as a minimum problem in this paper.

point. Reasonable value of step size Δr be such as to allow escape from a local minimum in a few (2-3) steps [11].

Since this algorithm is expected to approach to a global minimum and does not to want to escape from it as iteration proceeds, T and Δr are need to be adjusted to stay around the global minimum. Another reason that Δr should be reduced as iteration proceeds is that for a large value of Δr starting point x_0 may keep crossing over the point which corresponds to the global minimum. Therefore smaller step size is required to get closer to the global minimum.

Table I shows the pseudocode of the simulated annealing. With a fixed value of T and Δr, `N_eval` evaluation of the cost function is performed. This `N_eval` evaluation is called a "cycle". Temperature T is reduced after each cycle. And after reduction of T `Nt` times, Δr is reduced. After `Ns` reduction of Δr, iteration stops. It should be noted that each the filter coefficients of $\{h(k)\}$ should be inside $[-1, 1]$.

TABLE I
PSEUDOCODE OF SIMULATED ANNELAING

```
T = Temperature
Δr = Ns;
E0 = cost_function({h0(k)});
for s = 1 : Ns
    for t = 1 : Nt
        for l = 1 : N_eval
            {h1(k)} = random_walk({h0(k)}, Δr);
            E1 = cost_function({h1(k)});
            if E1 < E0
                {h1(k)} = {h0(k)};
                E1 = E0;
            else
                ΔE = E0 − E1;
                p = exp( −ΔE/T);
                ρ ∈ Uniform[0, 1];
                if p > ρ
                    accept : {h0(k)} = {h1(k)};
                             E0 = E1;
                else
                    reject : no action;
                end
            end
        end
        T = temp_adjustment(T);
    end
    T = Temperature;
    Δr = step_adjustment(Δr);
end
```

IV. Designing Methodology

This paper proposes new filter design method using simulated annealing. Filter coefficient update can be performed in two ways. Assume that object function is in the n-dimension hyper space(i.e. there are n-axis). Therefore, there are n valuables which are needed to be updated. Define $\{h_0(k)\}$ and $\{h_1(k)\}$ as the filter coefficients before update and after update, respectively. In addition, define E_0 and E_1 as the corresponding value of the cost function according to $\{h_0(k)\}$ and $\{h_1(k)\}$, respectively. If all the elements of $\{h_0(k)\}$ and $\{h_1(k)\}$ are different when the random walk is taken, it is called *block update*. On the other hand, if only one element of $\{h_0(k)\}$ and $\{h_1(k)\}$ is different, it is called *element-wise update*. Element wise update can be performed from axis 1 to axis n or regardless of order.

Since the block update updates all the filter coefficients at the same time, it is faster than element-wise update n times asymptotically for updating $\{h(k)\}$. However, element-wise update method has an advantage of capability of fine tuning. In other words, this method can find coefficients that has lower error. For example, if there is a need to update only one element of $\{h(k)\}$, block update method does not guarantee the reduction of cost function after update because block update method updates all the element of $\{h(k)\}$ at the same time. It result that starting point wanders around the optimal point. However, since element-wise update can adjust one element of $\{h(k)\}$, it can approach to the optimum point. Precision problem with block update can be lessened by reducing step size Δr.

Selecting starting point of $\{h(k)\}$ affects the overall iteration time as well as convergence accuracy since improper initial starting point selection leads to long iteration to converge and local minima depending on other design parameters. Therefore, initial point selection arise as an important issue. One solution is to use two-step simulated annealing which performs block update as an initial point selection step and performs element-wise update to solve an global optimization problem. Since block update as a first stage of the algorithm is fast and gives good initial guess, element wise update can approach to the optimal point with less iterations and improved accuracy at the second stage.

Sub-domain method can contribute to the speed up of designing. In the sub-domain method, range of the filter coefficient $[-1, 1]$ is broken into N_{sub} sub-domains. In the each sub-domain, same simulated annealing algorithm is performed to find optimal coefficients that minimize equation (2). Since sub-domain method searches optimum $\{h(k)\}$ within sub-domain which is usually smaller than entire domain $[-1, 1]$, operation time can be reduced with the help of parallel processing. And if sub-domain method is used, temperature T and step Δr can be smaller value when simulated annealing algorithm starts and smaller value of Nt, Ns, and N_eval are needed. These

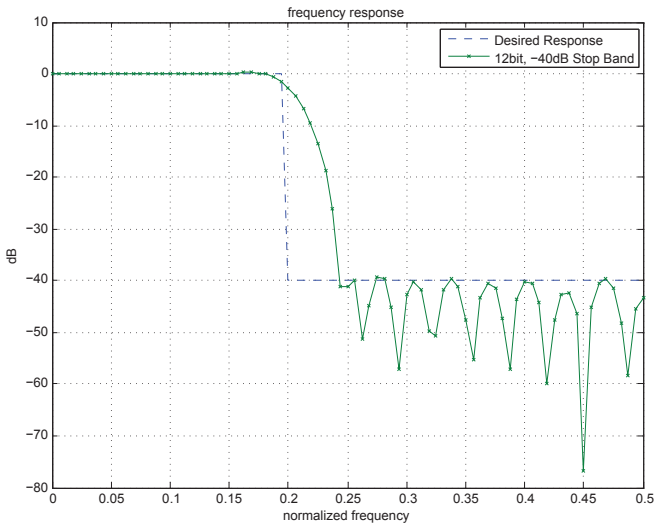

Fig. 1. Frequency Response

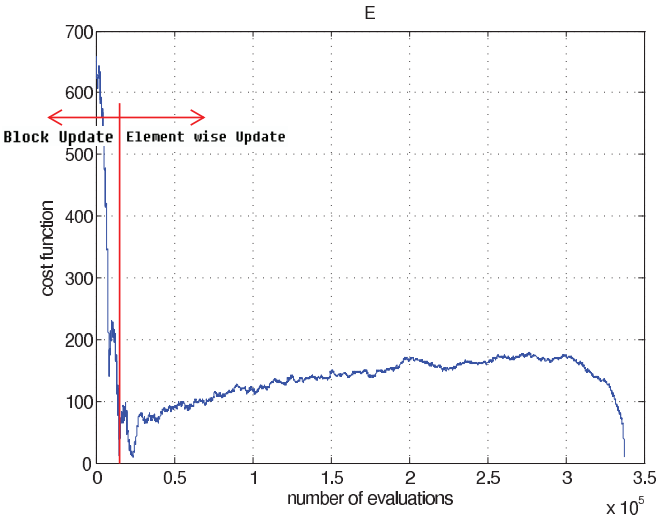

Fig. 2. Cost Function

also leads to speed-ups because smaller number of function evaluation. To achieve this kind of improvement multiple processing unit is needed to find filter coefficients of each domain parallel.

V. Design Example

This section provides a design example using two stage finite word length digital filter filter using simulated annealing.

Figures 1 is the frequency response of designed FIR filter. This figure is obtained using the algorithm specified in table I. Passband is $[0, 0.2]$, stopband is $[0.25, 0.5]$ and sidelobe level is $-40dB$. Each filter coefficient is composed of 12-bit. Number of step adjustment and temperature adjustment is 2 and 50 respectively. And initial temperature is 15. As can be seen the figure, frequency response shows the proper result with given parameters. By using Eq.9 stopband has equi-ripple sidelobe. This is the main characteristic of the design methodology of this paper.

Figure 2 is the value of cost function as iteration proceeds. This show the the two-step simulated annealing approach. Block update method, which is a initial point selection step, is up to the iteration number 200. In this step, initial point with relatively low cost function value is chosen. This step chose the initial point which is possibly close to global minimum by updating all the coefficients $\{h(k)\}$ at once. In the sequel, starting from this initial point selected by the first step, element wise update finds the new coefficients $\{h(k)\}$ by updating one element at each iteration. As can be seen, cost function decrease as iteration proceeds and block update gives very good initial values for $\{h(k)\}$. Even though there are periods that cost function increases, it eventually decreases to a certain value.

VI. Conclusion

This paper proposed a new finite wordlength FIR filter design algorithm using simulated annealing. By using two step algorithm, initial starting point can be selected deliberately at the first step which uses the block update. And second step which uses the element-wise update provide the result closer to the global minimum. With new algorithm, equi-ripple FIR filter can be designed. Design example proves the validity of this algorithm. Modified simulated annealing can be applied to many kinds of design problems not only to designing FIR filters.

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